Strain Based Approach to the Modeling of Concrete under Uniaxial Tension-Tension Fatigue Loading

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Abstract: **Strain based Approach to the Modeling of Concrete under Uniaxial Tension-Tension Fatigue Loading is briefly described regarding Stiffness degradation and inelastic deformation which are the essential features of concrete that develop due to the formation of multitude of microcracks in the fatigue environment. Microcracking, which is anisotropic in nature, destroys the bond between material grains, and affects the elastic properties resulting in the reduction of material stiffness. This paper presents Strain based Approach to the Modeling of Concrete under Uniaxial Tension-Tension Fatigue Loading. The model is developed, in strain space, using the general framework of internal variable theory of continuum thermodynamics. It is argued that within the damage surface of given strain states the unloading-reloading cycles (fatigue loading) stimulate the nucleation and growth microcracks in concrete, which will result in stiffness degradation and inelastic deformation, and hence material is termed as damaged. Damage is reflected through the fourth-order stiffness tensor involving a damage parameter whose increment is governed by the consistency equation associated with a cycle dependent damage surface in strain space. The model is capable of predicting stiffness degradation, inelastic deformation and strength reduction under fatigue loading and compared against experimental results.**

Keywords: **Fatigue; uniaxial; damage; Concrete; Thermodynamics; stiffness; microcracks, response tensors.**

1. INTRODUCTION

In recent times, concrete has become the bedrock of infrastructural civilization in Nepal. Statistics has shown that over 75% of the infrastructures in Nepal have to do with concrete. Thus, it is important to study and understand every aspect of concrete from the production, transportation, placing and eventually maintenance of concrete.

Concrete today has a very wide range of applications. Virtually every civil engineering work in Nepal today is directly or indirectly involving the use of concrete. The use of concrete in civil engineering works includes: construction of residential houses, industrial warehouses, roads pavement construction, Shore Protection works, piles, domes, bridges, culverts, drainages, canals, dams etc. (Shetty, 2005; Neville, 2011; Edward and David, 2009; Duggal, 2009; Gambhir, 2005). In Nepal, the cases of failure of structures and roads (concrete related failure) occur on a yearly basis.

Concrete is one of the most widely used materials in numerous civil engineering applications due to its workability and formability into various structural components. Except cement all ingredients of concrete are commonly available local materials like aggregate and water so concrete is getting more popular. Concrete is a composite material consisting of three components: the cement matrix, the aggregate and the interface between the matrix and aggregate. The cement-matrix is the weakest zone of the composite. It contains voids and micro cracks even before any load has been applied. A material or a component exposed to cyclic loading leads to increase of stress concentration around the micro cracks and finally leads to fracture. Forces that are required to obtain the fracture in cyclic loading are usually much less than forces that would have been required in case of monotonic loading. Phenomenon that deals with this type of fracture is called fatigue. It is caused by progressive, permanent internal structural changes in the material, which may result in micro cracks and their propagation until governing macro cracks are formed. The effect of cyclic loading is to develop permanent damage in the concrete materials as a result of which failure happens under the stress having value less than the ultimate strength of concrete. Concrete, a heterogeneous material comprising the mixture of cement, sand and aggregate, exhibits several mutually interacting inelastic mechanisms such as micro crack growth and inelastic flow even under small amplitude of cyclic load when applied in large number of cycles. As a consequence, concrete does not guarantee endurance fatigue limit like metal as described in Miner's hypothesis.

Reinforced concrete structures such as bridges, hydraulic foundations, pressure vessels, crane beams are subjected to long term cyclic loading. The effect of cyclic loading is to develop permanent damage in the concrete materials as a result of which failure happens under the stress having value less than the ultimate strength of concrete. Concrete, a heterogeneous material comprising the mixture of cement, sand and aggregate, exhibits several mutually interacting inelastic mechanisms such as microcrack growth and inelastic flow even under small amplitude of cyclic load when applied in large number of cycles. As a consequence, concrete does not guarantee endurance fatigue limit like metal as described in Miner's hypothesis [1].

The presence of permanent damage at fatigue failure has been documented by a number of investigations. [2] developed fatigue damage model for ordinary concrete subjected to cyclic compression based on mechanics of composite materials utilizing the concept of dual nature of fatigue damage, which are cycle dependent and time dependent damage. The model was capable of capturing the cyclic behavior of plain concrete due to progressive creep strain with the increase in number loading cycles. [3] used accelerated pavement testing results for carrying out cumulative fatigue damage analysis of concrete pavement. In [3], they reported that Miner hypothesis does not accurately predict cumulative fatigue damage in concrete. The experimental work of [4] clearly showed that increase of damage in the material takes place in about last 20% of its fatigue life. [5] presented a theoretical model to predict the fatigue process of concrete in alternate tension-compression fatigue loading using double bounding surface approach described in strain-energy release rate by constructing the damage-effective tensor.

In the past few years, a number of damage constitutive models have been published to model the observed mechanical behavior of concrete under monotonic and cyclic loading ([6], [7], [8], [9], and [10]). The need for such models arises from the physical observation that two dominant microstructural patterns of deformation in concrete are inelastic flow and microcracking. The inelastic flow component of deformations is modeled by using plasticity theories whereas the nucleation and propagation of microcracks and microvoids is incorporated in the constitutive models with the use of damage mechanics theories. The progressive development of cracks and microcracks alters the elastic properties (degradation of elastic moduli) due to energy dissipation and concrete material becomes more compliant.

This paper presents a class of damage mechanics theory to model the fatigue damage and failure of concrete caused by multitude of cracks and microcracks whereby anisotropic damaging behavior is captured through the use of proper response function involving damage parameter in material stiffness tensor. The increment of damage parameter is obtained from consistency equation in cycle dependent damage surface in strain space. The model is also capable of capturing the inelastic deformations that may arise due to misfits of crack surfaces and development of sizable crack tip process zone.

2. FORMULATION

In this paper, it is assumed that a continuum damage mechanics approach can be taken to describe the constitutive relation for concrete and that the fatigue loading is of low frequency so that the thermal effects could be ignored. For isothermal process, rate independent behavior and small deformations, the Helmholtz Free Energy (HFE) per unit volume can be deduced from [11] and shown as follows:

$$\mathrm{A}(\boldsymbol{\varepsilon},\mathrm{k}) = \tfrac{1}{2}\boldsymbol{\varepsilon} : \mathbf{E}(k) : \boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^{\mathbf{i}} : \boldsymbol{\varepsilon} + \mathrm{A}^{\mathrm{i}}(k) \quad .....(1) \qquad (1)$$

Where **E** (k) represents fourth order elastic stiffness tensor which depends on the state of microcracking (damage), $\boldsymbol{\varepsilon}$ is the strain tensor, $\boldsymbol{\sigma}^{\mathbf{i}}$ denotes the stress tensor corresponding to inelastic damage. The term $A^{i}(k)$ represents surface energy of microcracks [12], and k is the cumulative fatigue damage parameter. The colon (**:**) represents the tensor contraction operation.

For an inelastic damaging process, a constitutive relation between the stress and strain tensors can be established utilizing fourth order material's stiffness tensor as

$$\boldsymbol{\sigma} = \frac{\partial A}{\partial \boldsymbol{\varepsilon}} = \mathbf{E}(k) : \boldsymbol{\varepsilon} - \boldsymbol{\sigma}^{i}(k) \qquad .....(2)$$

The rate form of Eqn (2) with respect to cyclic number N is given by

$$\begin{aligned}\dot{\boldsymbol{\sigma}} &= \mathbf{E}(k) : \dot{\boldsymbol{\varepsilon}} + \dot{\mathbf{E}}(k) : \boldsymbol{\varepsilon} - \dot{\boldsymbol{\sigma}}^{i}(k) \\ &= \dot{\boldsymbol{\sigma}}^{\mathbf{e}} + \dot{\boldsymbol{\sigma}}^{\mathbf{D}}(k) + \dot{\boldsymbol{\sigma}}^{\mathbf{i}}(k)\end{aligned} \qquad .....(3)$$

Where $\dot{\boldsymbol{\sigma}}^{\mathbf{e}}$ is the stress increment in the absence of further damage in the material, $\dot{\boldsymbol{\sigma}}^{\mathbf{D}}$ is the rate of stress-relaxation due to further microcracking (elastic damage), and $\dot{\boldsymbol{\sigma}}^{\mathbf{i}}(k)$ designates the rate of stress tensor corresponding to irrecoverable or permanent deformation due to microcracking.

It is further assumed that damage during fatigue loading alters elastic properties and affects the stiffness tensor. For small deformation, the following decomposition of the fourth-order stiffness tensor, E, is adopted

$$\frac{\partial^2 A}{\partial \boldsymbol{\varepsilon} \partial \boldsymbol{\varepsilon}} = \mathbf{E}(k) = \mathbf{E}^{\mathbf{0}} + \mathbf{E}^{\mathbf{D}}(k) \qquad .....(4)$$

Where $\mathbf{E}^{\mathbf{0}}$ is stiffness of the undamaged or virgin material and $\mathbf{E}^{\mathbf{D}}(k)$ denotes the overall stiffness degradation caused by damage during fatigue loadings. Further, $\dot{\mathbf{E}}(k)$ and $\dot{\boldsymbol{\sigma}}^{\mathbf{i}}(k)$ are expressed as fluxes in the thermodynamic state sense and are expressed in terms of evolutionary equations as

$$\dot{\mathbf{E}}^{\mathbf{D}} = -\dot{\mathrm{k}}\mathbf{L} \quad \text{and} \quad \dot{\boldsymbol{\sigma}}^{i} = \dot{\mathrm{k}}\mathbf{M} \qquad .....(5)$$

Where **L** and **M** are, respectively, fourth and second order response tensors that determine the directions of the elastic and inelastic damage processes. Following the Clausius-Duhem inequality, utilizing the standard thermodynamic arguments [13] and assuming that the unloading is an elastic process, a potential function

$$\Psi(\boldsymbol{\varepsilon}, k) = \tfrac{1}{2}\boldsymbol{\varepsilon} : \mathbf{L} : \boldsymbol{\varepsilon} - \mathbf{M} : \boldsymbol{\varepsilon} - \tfrac{1}{2} p^2(\boldsymbol{\varepsilon}, k) = 0 \quad .....(6)$$

is obtained as damage surface which establishes the onset of material inelasticity and stiffness deterioration. In Eqn (6), $p(\boldsymbol{\varepsilon}, k)$ is interpreted as the damage function given below as

$$p^2(\boldsymbol{\varepsilon}, k) = 2\left[h^2(\boldsymbol{\varepsilon}, k) + \frac{\partial A^i}{\partial k}\right] \qquad .....(7)$$

for some scalar valued function $h^2(\boldsymbol{\varepsilon}, k)$. It should be noted that as long as the function $p^2(\boldsymbol{\varepsilon}, k)$ is defined, the individual terms on the R.H.S of Eqn (7) need not to be identified.

To progress further, specific forms of response tensors **L** and **M** must be specified. Since damage is highly directional, the response tensors should be formulated to address such directionality. In determining a proper form for the response tensor, the strain tensor is decomposed into positive and negative cones. The positive and negative cones of the strain tensor hold the corresponding positive and negative eigenvalue of the system, that is, $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{+} + \boldsymbol{\varepsilon}^{-}$. Here, $\boldsymbol{\varepsilon}^{+}$ and $\boldsymbol{\varepsilon}^{-}$ represent the positive and negative cones of the strain tensor, respectively. Based on the experimental observations and results for concrete materials, damage is assumed to occur in the cleavage mode of cracking as shown schematically in Figure 1.

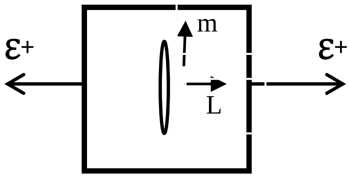

Figure 1. Schematic representation of mode I damage (crack opening in tensile mode) under cyclic tension.

For cleavage cracking mode, the following forms of response tensors are postulated for **L** and **M**

$$\mathbf{L} = \frac{\boldsymbol{\varepsilon}^{+} \otimes \boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}} \qquad .....(8)$$

$$\mathbf{M} = \beta \boldsymbol{\varepsilon}^{+} \qquad .....(9)$$

The substitution of response tensors **L** and **M** from Eqns (8) and (9) into Eqn (6) leads to the final form of the damage surface

$$\Psi(\boldsymbol{\varepsilon}, k) = \tfrac{1}{2} \boldsymbol{\varepsilon} : \frac{\boldsymbol{\varepsilon}^{+} \otimes \boldsymbol{\varepsilon}^{+}}{\boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}} : \boldsymbol{\varepsilon} - \beta \boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon} - \tfrac{1}{2} p^{2}(\boldsymbol{\varepsilon}, k) = 0 \qquad ....(10a)$$

For uniaxial tensile loading the damage surface of Eqn (10a) is rewritten as

$$\Psi(\boldsymbol{\varepsilon}, k) = \tfrac{1}{2} \boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+} - \beta \boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+} - \tfrac{1}{2} p^{2}(\boldsymbol{\varepsilon}, k) = 0$$
$$= \tfrac{1}{2} \boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}(1 - 2\beta) - \tfrac{1}{2} p^{2}(\boldsymbol{\varepsilon}, k) = 0 \; ...(10b)$$

The damage function p(k) obtained from uniaxial tensile test for concrete materials based on the experimental results of [14] is given in [15] as

$$\mathrm{p}(k) = \varepsilon_{u} \, ln\left(\frac{\mathrm{E}^{0}}{\mathrm{E}^{0} - \mathrm{k}}\right) \qquad .....(11)$$

And for elastic damaging process, ($\beta = 0$), the limit damage surface reduces to

$$\mathrm{p}(k) = \varepsilon_{u} \qquad .....(12)$$

Where ε_{u} represents the strain corresponding to uniaxial tensile strength of concrete, which is used as the reference strain and hence the result of a conventional uniaxial tensile test is needed to establish ε_{u} .

3. FATIGUE DAMAGE MODEL

Fatigue loading is understood as the repeated or fluctuating strains acting in a material due to which progressive permanent structural change occurs in the form of cracks or flaws and the material fails at stresses having a maximum value less than the tensile strength of the material. In this paper, it is assumed that within the damage surface of the given strain state, the unloading-reloading cycles (Fatigue loadings) increase damage in concrete due to the growth of microcracks leading to inelastic deformations and stiffness degradation, which eventually reduces the ultimate strength of the concrete. To achieve this, the damage surface $\Psi(\boldsymbol{\varepsilon}, k)$ is modified to predict the increase in damage in the material with increasing number of cycles of loading as

$$\tfrac{1}{2} \boldsymbol{\varepsilon}^{+} : \boldsymbol{\varepsilon}^{+}(1 - 2\beta) X(N) - \tfrac{1}{2} p^{2}(\boldsymbol{\varepsilon}, k) = 0 \qquad .....(13)$$

Where X(N) is a function that depends on the number of loading cycles and adopted to increase of damage with increasing number of cycles. We propose a power function for X(N) as

$$X(N) = N^{A} \qquad .....(14)$$

Here, N represents the number of loading cycles, and A is a material parameter. Utilizing Eqns (11) through (14), we obtain the cumulative fatigue parameter k as

$$k = E^0\left[1 - \frac{1}{exp\left(\frac{\sqrt{(1-2\beta)N^A \boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+}}{\varepsilon_u}\right)}\right] \quad ...(15)$$

By differentiating Eqn (15) with respect to N, the increment of damage in one cycle is obtained as

$$\dot{k} = \frac{dk}{dN}$$

$$= \frac{AN^{\frac{A}{2}-1} E^0 \sqrt{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+ (1-2\beta)}}{2\varepsilon_u exp(-\sqrt{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+ (1-2\beta) N^A / \varepsilon_u^2})} \quad (16)$$

Finally, the rate of damage parameter, $\dot{k}$, must be used in the simple constitutive relation of the form given by Eqn (3) in uniaxial tensile stress path for representing inelastic deformation, stiffness degradation and strength reduction due to fatigue cycles. Substituting Eqns (8), (9) and (16) into Eqn (5) and then substituting Eqns (4) and (5) into Eqn (3) yields

$$\dot{\boldsymbol{\sigma}} = \mathbf{E}(k) : \dot{\boldsymbol{\varepsilon}} - \dot{k}\left(\frac{\boldsymbol{\varepsilon}^+ \otimes \boldsymbol{\varepsilon}^+}{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+} : \boldsymbol{\varepsilon} + \beta \boldsymbol{\varepsilon}^+\right) \quad .....(17)$$

Eqn (17) is the rate of stress tensor for uniaxial tension-tension fatigue loading where it is assumed that unloading is elastic process. When $\beta = 0$, the process is classified as elastic-damaging, in which stress-strain curve returns to origin upon unloading of the material. In fact, damage incurred in concrete cannot be considered perfectly elastic. The unloaded material shows some residual strains due to the development of sizable crack tip process zone and misfits of the crack surfaces.

For uniaxial tension, Eqn (17) becomes

$$\dot{\boldsymbol{\sigma}} = \mathbf{E} : \dot{\boldsymbol{\varepsilon}} - \left[\frac{AN^{\frac{A}{2}-1} E^0 \sqrt{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+ \eta}((1+\beta))}{2\varepsilon_u exp\left(-\sqrt{\frac{\boldsymbol{\varepsilon}^+ : \boldsymbol{\varepsilon}^+ \eta N^A}{\varepsilon_u^2}}\right)}\right] \boldsymbol{\varepsilon}^+ \quad ...(18)$$

Where, $\eta = 1 - 2\beta$.

4. NUMERICAL EXAMPLES

The proposed model contains two material parameters, first is A which is factor related to materials intermolecular microcracks and second one is β which is called damage factor related to kinematic phenomena of the particle i.e. crack surface close perfectly after unloading. The damage parameter, k, which is regarded as the stiffness degradation in the proposed model, is computed by measuring stiffness at three different stages of loading cycle. The kinematic parameter, β, is determined by measuring the permanent deformation during one of the cyclic loadings. Due to the scarcity of experimental data in the literature for the measurement of these material parameters in the numerical simulation, analyst's judgments are required to obtain numerical results.

The model prediction of modulus reduction with increasing number of cyclic loading is shown in Figure **3**. Figure **(6)** and **(7)** shows the decrease of maximum stress level (S-N curve) in cyclic tension-tension loading. Figure **(10)** and **(11)** on other hand, shows corresponding experimental result regarding decrease of materials stress and increase of cumulative fatigue damage parameter with respect to increase of number of cyclic loading. As may be seen, the model captures the relevant features of cyclic response.

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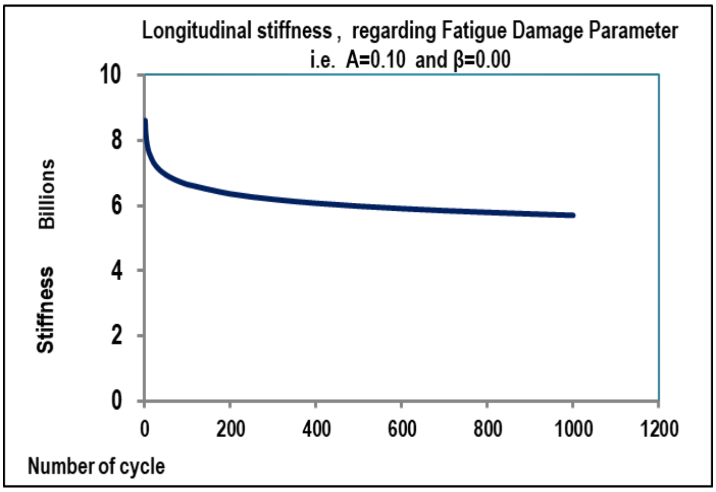

Figure 2. Model prediction of stiffness reduction with number of cyclic loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.00

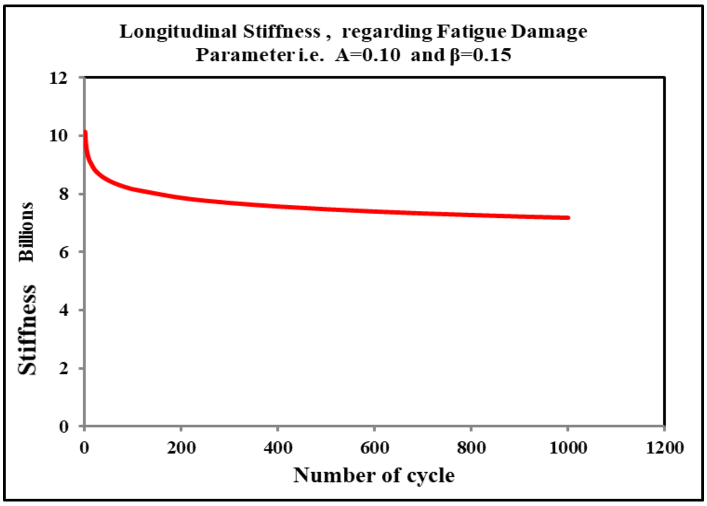

Figure 3. Model prediction of stiffness reduction with number of cyclic loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.15

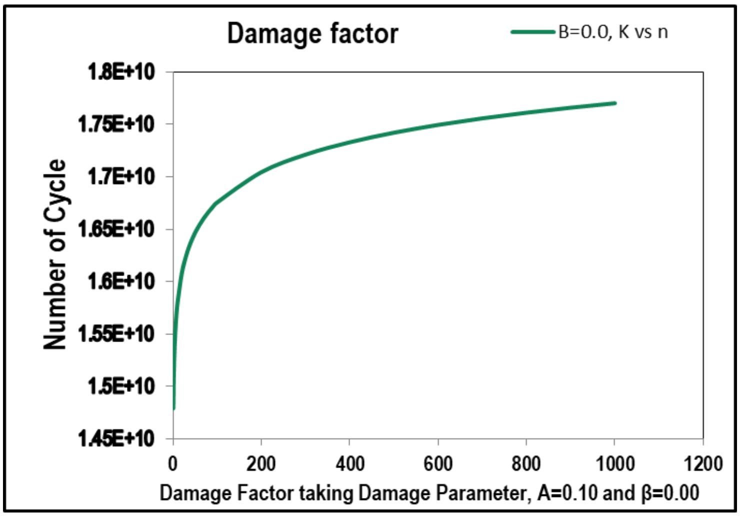

Figure 4. Model prediction of Damage Factor with number of cyclic loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.00

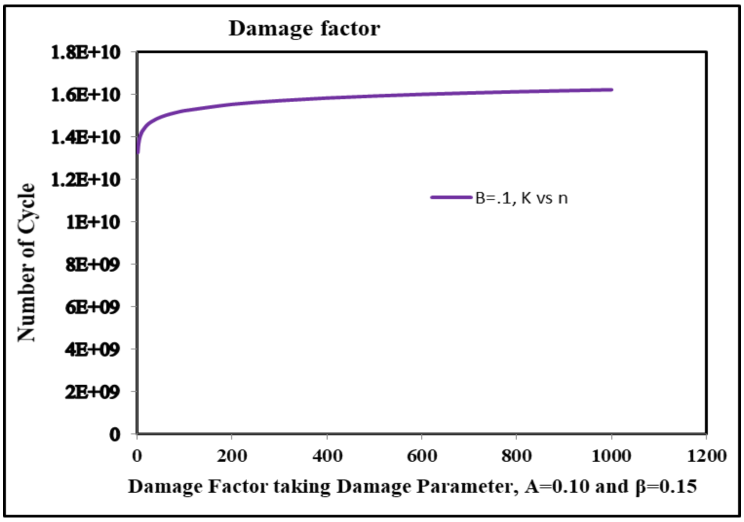

Figure 5. Model prediction of Damage Factor with number of cyclic loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.15

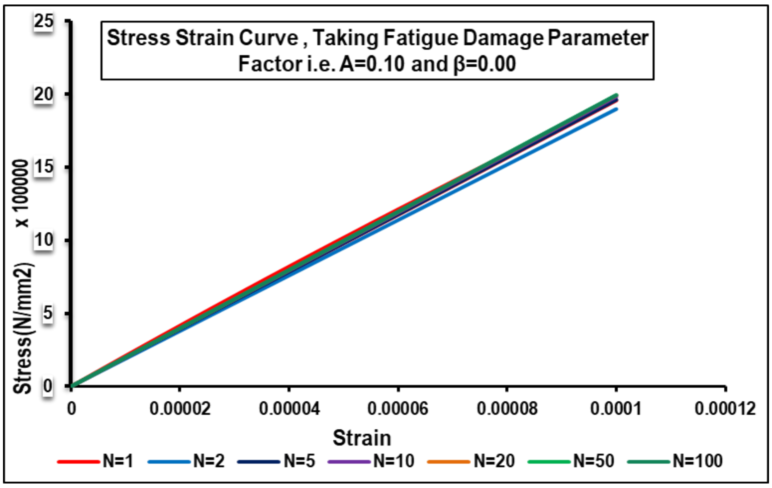

Figure 6. Theoretical cyclic stress-strain behavour of concrete during elastic damaging process under uniaxial fatigue loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.00

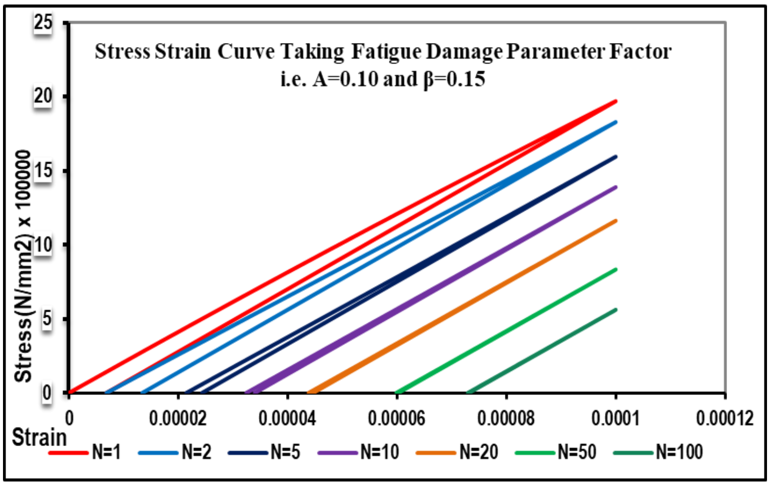

Figure 7. Theoretical cyclic stress-strain behavour of concrete during elastic damaging process under uniaxial fatigue loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β= 0.15

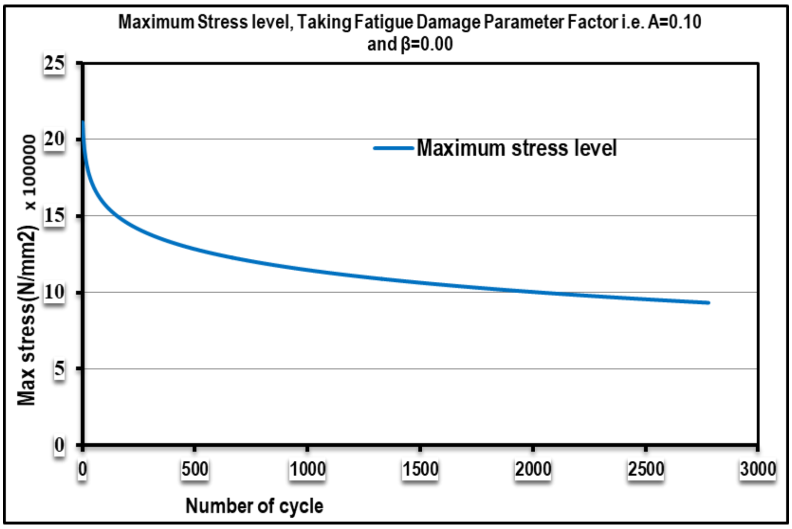

Figure 8. Model prediction of Maximum Stress Level verses Number of Cycle under uniaxial fatigue loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.00

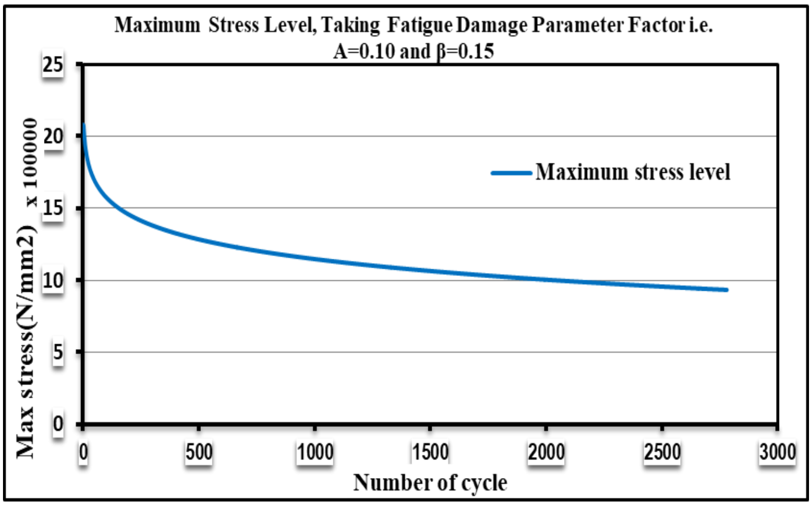

Figure 9. Model prediction of Maximum Stress Level verses Number of Cycle under uniaxial fatigue loading. Adopting the Value of Fatigue Damage Parameter, A=0.10 and β = 0.15

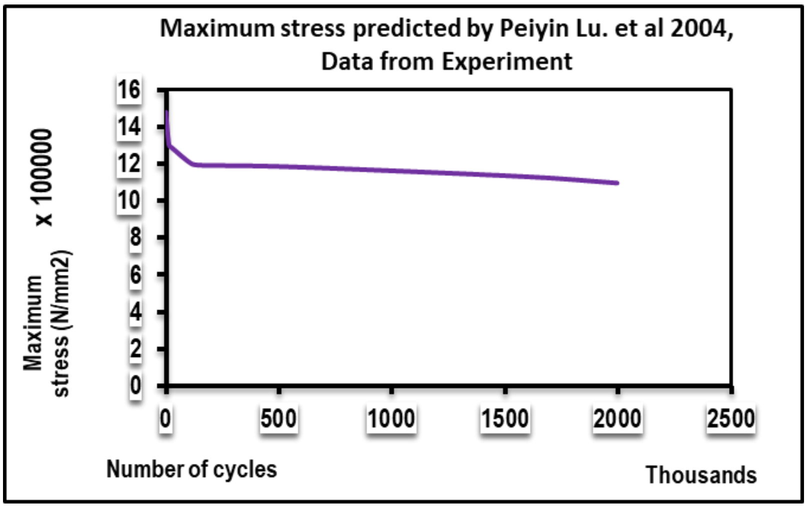

Figure 10. Reduction in maximum stress level during cyclic tension. Prediction of the theory, Figure 8 and 9 by Peiyin Lu. Et. Al 2004

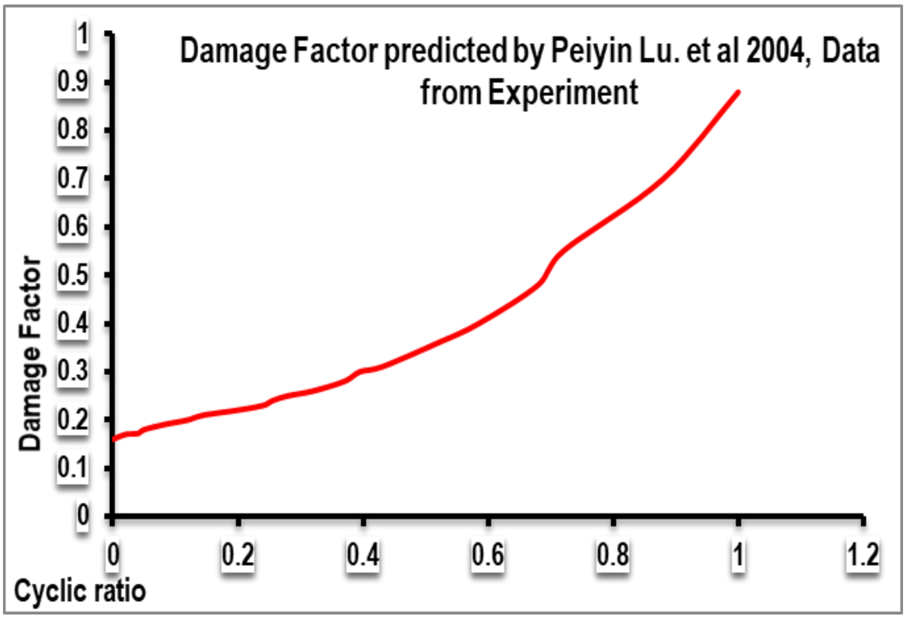

Figure 11. Variation of damage with the number of cyclic loading. Prediction of the theory, Figure (4) and Figure (5). Experimental Figure [11], by Peiyin Lu. Et. Al 2004

Figure **(4) and (5)** shows the increase in damage with increasing loading cycles. The experimental work of Figure [11] is also shown for comparison. The theoretical model also captures the similar trend of increasing damage with the cyclic loading as observed in the experiment [11]. For numerical simulation, the following constant were used, A = 0.10 and β = 0.15 and 0.00 in two cases, Parameter A is estimated by comparing predicted results and experimental results over a range of applied strains.

Figures (6) and **(7)** depict the theoretical cyclical stress-strain behavior of concrete material in tension. In Figure **6**, no permanent deformations are predicted to remain upon unloading of concrete material however progressive damage accumulation takes place in each loading cycle due to degradation of elastic modulus. This is the ideal case of elastic perfectly damaging behavior which can be achieved by letting $\beta = 0$ with the assumption that crack surfaces close perfectly upon unloading. Heterogeneous materials like concrete exhibits permanent deformations. Figure 7 shows the versatile behavior of the model where the stiffness degradation and permanent deformation are illustrated simultaneously.

5. CONCLUSION

Strain Based Approach to the Modeling of Concrete under Uniaxial Tension-Tension Fatigue Loading of concrete materials during low frequency is presented by utilizing the framework of continuum thermodynamics of Continuum Mechanics by taking two material fatigue damage parameter i.e. A=fatigue damage Parameter regarding energy microcracks of the material particle and another is β=kinematic damage Parameter (phenomena of material crack surface close perfectly after unloading). Since the fatigue damage in concrete during the fatigue process is mainly due to development of microcracks and microvoids, a cycle dependend damage surface is employed in the formulation of this strain based theoretical model. Fatigue damage evolution law together with the damage response functions were used in the constitutive relation to demonstrate the capability of the model in capturing the essential features of concrete material, such as stiffness degradation and the inelastic deformations, under fatigue loading environment by finding out the cumulative fatigue damage parameter i.e. K. The fatigue curve relavant to A =0.10 and β =0.15 and 0.00 is generated by the modeling and after that this generated model curve is compared to the Curve obtained by Peiyin Lu. Et al (2004) which shows similar tread of generation of fatigue curve. This shows the good relation between results obtained from modeling and experiments also. Lower value in the experimental curve is due to 0.85 times maximum stress level whereas, modeling takes 100% value.

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APPENDICES-A

List of Table:

Table 1. Data Analysis for Fatigue Curve regarding Stiffness, Cumulative Damage Parameter Verses Number of Cycle of Concrete

Assuming , Fatigue Damage Factor related to Surface Energy Microcracks i.e. A= 0.10 and Fatigue Kinematic Damage Factor (Crack Surface closed perfectly after unloading) i.e. β=0.00

N (Number of Cycle)	Fatigue Damage Parameter (A)	Stiffness factor (β)	E0	εu	Exp(sqrt((1-2xβ)xN^Axεuxεu)/εu)	(E0-k)/E0	(Eo-k)	1-((E0-k)/E0)	Fatigue Damage Parameter, K
1	0.1	0.00	2.34E+10	1E-04	2.718281828	0.367879	8608378923	0.632120559	14791621077
2	0.1	0.00	2.34E+10	1E-04	2.815852123	0.355132	8310095480	0.644867715	15089904520
3	0.1	0.00	2.34E+10	1E-04	2.876192321	0.347682	8135756372	0.652318104	15264243628
4	0.1	0.00	2.34E+10	1E-04	2.920554404	0.342401	8012177404	0.657599256	15387822596
5	0.1	0.00	2.34E+10	1E-04	2.955885852	0.338308	7916408539	0.661691943	15483591461
6	0.1	0.00	2.34E+10	1E-04	2.985369569	0.334967	7838225541	0.665033097	15561774459
7	0.1	0.00	2.34E+10	1E-04	3.010740403	0.332144	7772174570	0.667855788	15627825430
8	0.1	0.00	2.34E+10	1E-04	3.0330523	0.329701	7715000497	0.670299124	15684999503
9	0.1	0.00	2.34E+10	1E-04	3.052995299	0.327547	7664604006	0.67245282	15735395994
10	0.1	0.00	2.34E+10	1E-04	3.071046719	0.325622	7619551945	0.674378122	15780448055
20	0.1	0.00	2.34E+10	1E-04	3.194997641	0.312989	7323949068	0.687010724	16076050932
30	0.1	0.00	2.34E+10	1E-04	3.271916231	0.305631	7151772341	0.694368703	16248227659
40	0.1	0.00	2.34E+10	1E-04	3.328592706	0.300427	7029997981	0.699572736	16370002019
50	0.1	0.00	2.34E+10	1E-04	3.373807035	0.296401	6935784933	0.703598934	16464215067
60	0.1	0.00	2.34E+10	1E-04	3.411588452	0.293119	6858975029	0.706881409	16541024971
70	0.1	0.00	2.34E+10	1E-04	3.444135968	0.290349	6794156857	0.709651416	16605843143
80	0.1	0.00	2.34E+10	1E-04	3.472786939	0.287953	6738104126	0.712046832	16661895874
90	0.1	0.00	2.34E+10	1E-04	3.498417763	0.285844	6688738048	0.714156494	16711261952
100	0.1	0.00	2.34E+10	1E-04	3.521635146	0.283959	6644640638	0.716040998	16755359362
200	0.1	0.00	2.34E+10	1E-04	3.681503839	0.271628	6356098221	0.728371871	17043901779
300	0.1	0.00	2.34E+10	1E-04	3.781094589	0.264474	6188684110	0.73552632	17211315890
400	0.1	0.00	2.34E+10	1E-04	3.854660161	0.259426	6070574065	0.740573758	17329425935
500	0.1	0.00	2.34E+10	1E-04	3.913457797	0.255528	5979366896	0.7444715	17420633104
600	0.1	0.00	2.34E+10	1E-04	3.962663371	0.252356	5905119312	0.747644474	17494880688
700	0.1	0.00	2.34E+10	1E-04	4.005105817	0.249681	5842542263	0.750318707	17557457737
800	0.1	0.00	2.34E+10	1E-04	4.042507649	0.247371	5788486265	0.752628792	17611513735
900	0.1	0.00	2.34E+10	1E-04	4.075998823	0.245339	5740924131	0.754661362	17659075869
1000	0.1	0.00	2.34E+10	1E-04	4.106362272	0.243525	5698474331	0.756475456	17701525669

Table 2. Data Analysis for Fatigue Curve regarding Stiffness, Cumulative Damage Parameter Verses Number of Cycle of Concrete

Assuming , Fatigue Damage Factor related to Surface Energy Microcracks i.e. A= 0.10 and Fatigue Kinematic Damage Factor (Crack Surface closed perfectly after unloading) i.e. β=0.15

N (Number of Cycle)	Fatigue Damage Parameter (A)	Stiffness factor (β)	E0	εu	Exp(sqrt((1-2xβ)xN^Axεuxεu)/εu)	(E0-k)/E0	(Eo-k)	1-((E0-k)/E0)	Fatigue Damage Parameter, K
1	0.1	0.15	2.34E+10	1E-04	2.308643278	0.433155	10135823157	0.566845164	13264176843
2	0.1	0.15	2.34E+10	1E-04	2.377774054	0.420561	9841136907	0.579438594	13558863093
3	0.1	0.15	2.34E+10	1E-04	2.420330174	0.413167	9668102415	0.58683323	13731897585
4	0.1	0.15	2.34E+10	1E-04	2.451524347	0.407909	9545081625	0.592090529	13854918375
5	0.1	0.15	2.34E+10	1E-04	2.476313066	0.403826	9449532179	0.596173839	13950467821
6	0.1	0.15	2.34E+10	1E-04	2.496961949	0.400487	9371388303	0.59951332	14028611697
7	0.1	0.15	2.34E+10	1E-04	2.514703724	0.397661	9305271146	0.60233884	14094728854
8	0.1	0.15	2.34E+10	1E-04	2.530286213	0.395212	9247965658	0.604787792	14152034342
9	0.1	0.15	2.34E+10	1E-04	2.544198437	0.393051	9197395793	0.606948898	14202604207
10	0.1	0.15	2.34E+10	1E-04	2.556778301	0.391117	9152142754	0.608882788	14247857246
20	0.1	0.15	2.34E+10	1E-04	2.642836748	0.378381	8854122380	0.621618702	14545877620
30	0.1	0.15	2.34E+10	1E-04	2.695965947	0.370925	8679634854	0.629075434	14720365146
40	0.1	0.15	2.34E+10	1E-04	2.7349829	0.365633	8555812178	0.634367001	14844187822
50	0.1	0.15	2.34E+10	1E-04	2.766031321	0.361529	8459774053	0.638471194	14940225947
60	0.1	0.15	2.34E+10	1E-04	2.791923492	0.358176	8381318496	0.641823996	15018681504
70	0.1	0.15	2.34E+10	1E-04	2.814191249	0.355342	8314999917	0.644658123	15085000083
80	0.1	0.15	2.34E+10	1E-04	2.833764703	0.352887	8257566330	0.64711255	15142433670
90	0.1	0.15	2.34E+10	1E-04	2.851252541	0.350723	8206919472	0.649276946	15193080528
100	0.1	0.15	2.34E+10	1E-04	2.867075637	0.348787	8161626327	0.65121255	15238373673
200	0.1	0.15	2.34E+10	1E-04	2.97557367	0.33607	7864029796	0.663930351	15535970204
300	0.1	0.15	2.34E+10	1E-04	3.042772678	0.328648	7690354317	0.67135238	15709645683
400	0.1	0.15	2.34E+10	1E-04	3.092225392	0.323392	7567365582	0.676608308	15832634418
500	0.1	0.15	2.34E+10	1E-04	3.131639867	0.319322	7472123551	0.680678481	15927876449
600	0.1	0.15	2.34E+10	1E-04	3.164550061	0.316001	7394416125	0.683999311	16005583875
700	0.1	0.15	2.34E+10	1E-04	3.192883267	0.313197	7328799095	0.686803457	16071200905
800	0.1	0.15	2.34E+10	1E-04	3.217810885	0.31077	7272024626	0.689229717	16127975374
900	0.1	0.15	2.34E+10	1E-04	3.240100166	0.308632	7221998952	0.691367566	16178001048
1000	0.1	0.15	2.34E+10	1E-04	3.260282016	0.306722	7177293218	0.693278068	16222706782

Table 3. Data Analysis for Fatigue Curve regarding Stress Verses Strain

Assuming , Fatigue Damage Factor related to Surface Energy Microcracks i.e. A= 0.10 and Fatigue Kinematic Damage Factor (Crack Surface closed perfectly after unloading) i.e. β=0.00 at different Number of Cycle

N	E	Strain	dK	K	$stress/cycle	Ex Strain + (Stress/Cycle) /50	stress
1	23400000000	0	0	0	-225991.2969	42280.17406	0
1	23336400000	0.000002	3.18E+09	63600000	-225991.2969	42152.97406	42280.17406
1	23272800000	0.000004		63600000	-225991.2969	42025.77406	84433.14812
1	23209200000	0.000006		63600000	-225991.2969	41898.57406	126458.9222
1	23145600000	0.000008		63600000	-225991.2969	41771.37406	168357.4962
1	23082000000	0.00001		63600000	-225991.2969	41644.17406	210128.8703
1	23018400000	0.000012		63600000	-225991.2969	41516.97406	251773.0444
1	22954800000	0.000014		63600000	-225991.2969	41389.77406	293290.0184
1	22891200000	0.000016		63600000	-225991.2969	41262.57406	334679.7925
1	22827600000	0.000018		63600000	-225991.2969	41135.37406	375942.3666
1	22764000000	0.00002		63600000	-225991.2969	41008.17406	417077.7406
1	22700400000	0.000022		63600000	-225991.2969	40880.97406	458085.9147
1	22636800000	0.000024		63600000	-225991.2969	40753.77406	498966.8887
1	22573200000	0.000026		63600000	-225991.2969	40626.57406	539720.6628
1	22509600000	0.000028		63600000	-225991.2969	40499.37406	580347.2369
1	22446000000	0.00003		63600000	-225991.2969	40372.17406	620846.6109
1	22382400000	0.000032		63600000	-225991.2969	40244.97406	661218.785
1	22318800000	0.000034		63600000	-225991.2969	40117.77406	701463.759
1	22255200000	0.000036		63600000	-225991.2969	39990.57406	741581.5331
1	22191600000	0.000038		63600000	-225991.2969	39863.37406	781572.1072
1	22128000000	0.00004		63600000	-225991.2969	39736.17406	821435.4812
1	22064400000	0.000042		63600000	-225991.2969	39608.97406	861171.6553
1	22000800000	0.000044		63600000	-225991.2969	39481.77406	900780.6294
1	21937200000	0.000046		63600000	-225991.2969	39354.57406	940262.4034
1	21873600000	0.000048		63600000	-225991.2969	39227.37406	979616.9775
1	21810000000	0.00005		63600000	-225991.2969	39100.17406	1018844.352
1	21746400000	0.000052		63600000	-225991.2969	38972.97406	1057944.526
1	21682800000	0.000054		63600000	-225991.2969	38845.77406	1096917.5
1	21619200000	0.000056		63600000	-225991.2969	38718.57406	1135763.274
1	21555600000	0.000058		63600000	-225991.2969	38591.37406	1174481.848
1	21492000000	0.00006		63600000	-225991.2969	38464.17406	1213073.222
1	21428400000	0.000062		63600000	-225991.2969	38336.97406	1251537.396
1	21364800000	0.000064		63600000	-225991.2969	38209.77406	1289874.37
1	21301200000	0.000066		63600000	-225991.2969	38082.57406	1328084.144
1	21237600000	0.000068		63600000	-225991.2969	37955.37406	1366166.718
1	21174000000	0.00007		63600000	-225991.2969	37828.17406	1404122.092
1	21110400000	0.000072		63600000	-225991.2969	37700.97406	1441950.266

Table 4. Data Analysis for Fatigue Curve regarding Stress Verses Strain							
Assuming , Fatigue Damage Factor related to Surface Energy Microcracks i.e. A= 0.10 and Fatigue Kinematic Damage Factor (Crack Surface closed perfectly after unloading) i.e. β=0.15 at different Number of Cycle							
N	E	Strain	dK	K	$stress/cycle	Ex Strain + (Stress/Cycle)/50	stress
1	23400000000	0	0	0	-259889.9915	41602.20017	0
1	23354801741	0.000002	2.26E+09	45198259.4	-259889.9915	41511.80365	41602.20017
1	23309603481	0.000004		45198259.4	-259889.9915	41421.40713	83114.00382
1	23264405222	0.000006		45198259.4	-259889.9915	41331.01061	124535.411
1	23219206962	0.000008		45198259.4	-259889.9915	41240.6141	165866.4216
1	23174008703	0.00001		45198259.4	-259889.9915	41150.21758	207107.0357
1	23128810444	0.000012		45198259.4	-259889.9915	41059.82106	248257.2532
1	23083612184	0.000014		45198259.4	-259889.9915	40969.42454	289317.0743
1	23038413925	0.000016		45198259.4	-259889.9915	40879.02802	330286.4988
1	22993215666	0.000018		45198259.4	-259889.9915	40788.6315	371165.5269
1	22948017406	0.00002		45198259.4	-259889.9915	40698.23498	411954.1584
1	22902819147	0.000022		45198259.4	-259889.9915	40607.83846	452652.3933
1	22857620887	0.000024		45198259.4	-259889.9915	40517.44195	493260.2318
1	22812422628	0.000026		45198259.4	-259889.9915	40427.04543	533777.6738
1	22767224369	0.000028		45198259.4	-259889.9915	40336.64891	574204.7192
1	22722026109	0.00003		45198259.4	-259889.9915	40246.25239	614541.3681
1	22676827850	0.000032		45198259.4	-259889.9915	40155.85587	654787.6205
1	22631629590	0.000034		45198259.4	-259889.9915	40065.45935	694943.4763
1	22586431331	0.000036		45198259.4	-259889.9915	39975.06283	735008.9357
1	22541233072	0.000038		45198259.4	-259889.9915	39884.66631	774983.9985
1	22496034812	0.00004		45198259.4	-259889.9915	39794.2698	814868.6648
1	22450836553	0.000042		45198259.4	-259889.9915	39703.87328	854662.9346
1	22405638294	0.000044		45198259.4	-259889.9915	39613.47676	894366.8079
1	22360440034	0.000046		45198259.4	-259889.9915	39523.08024	933980.2847
1	22315241775	0.000048		45198259.4	-259889.9915	39432.68372	973503.3649
1	22270043515	0.00005		45198259.4	-259889.9915	39342.2872	1012936.049
1	22224845256	0.000052		45198259.4	-259889.9915	39251.89068	1052278.336
1	22179646997	0.000054		45198259.4	-259889.9915	39161.49416	1091530.227
1	22134448737	0.000056		45198259.4	-259889.9915	39071.09765	1130691.721
1	22089250478	0.000058		45198259.4	-259889.9915	38980.70113	1169762.818
1	22044052218	0.00006		45198259.4	-259889.9915	38890.30461	1208743.519
1	21998853959	0.000062		45198259.4	-259889.9915	38799.90809	1247633.824
1	21953655700	0.000064		45198259.4	-259889.9915	38709.51157	1286433.732
1	21908457440	0.000066		45198259.4	-259889.9915	38619.11505	1325143.244
1	21863259181	0.000068		45198259.4	-259889.9915	38528.71853	1363762.359
1	21818060922	0.00007		45198259.4	-259889.9915	38438.32201	1402291.077
1	21772862662	0.000072		45198259.4	-259889.9915	38347.92549	1440729.399

Table 5. Data Analysis for Fatigue Curve regarding Maximum Stress Verses Number of Cycle and Damage Factor Parameter Verses Number of Cycle

Assuming , Fatigue Damage Factor related to Surface Energy Microcracks i.e. A= 0.10 and Fatigue Kinematic Damage Factor (Crack Surface closed perfectly after unloading) i.e. β=0.00 at different Number of Cycle

N	$K	K	E	$stress	maximim stress	resudial strain	Log(N)
1	1282173402	0	23400000000	2114008.7	2114008.703	0.000204421	0
2	670041551	1282173402	22117826598	2091299.33	2091299.328	0.000202493	0.30103
3	458458692	1952214953	21447785047	2061344.27	2061344.267	0.000201791	0.477121
4	350269622	2410673645	20989326355	2034632.17	2034632.172	0.000201418	0.60206
5	284280732	2760943267	20639056733	2011362.18	2011362.182	0.000201185	0.69897
6	239711345	3045223999	20354776001	1990921.92	1990921.916	0.000201023	0.778151
7	207530075	3284935345	20115064655	1972746.93	1972746.93	0.000200905	0.845098
8	183169905	3492465419	19907534581	1956400.11	1956400.112	0.000200813	0.90309
9	164069326	3675635324	19724364676	1941550.91	1941550.908	0.00020074	0.954243
10	148678707	3839704650	19560295350	1927947.54	1927947.54	0.000200681	1
11	136004711	3988383357	19411616643	1915395.34	1915395.344	0.000200631	1.041393
12	125380779	4124388068	19275611932	1903741.33	1903741.327	0.000200589	1.079181
13	116342712	4249768847	19150231153	1892863.35	1892863.347	0.000200553	1.113943
14	108557141	4366111560	19033888440	1882662.48	1882662.476	0.000200522	1.146128
15	101778373	4474668701	18925331299	1873057.53	1873057.529	0.000200494	1.176091
16	95821285.7	4576447074	18823552926	1863981.1	1863981.099	0.000200469	1.20412
17	90543702.5	4672268359	18727731641	1855376.62	1855376.622	0.000200448	1.230449
18	85834594.3	4762812062	18637187938	1847196.18	1847196.182	0.000200428	1.255273
19	81605972.8	4848646656	18551353344	1839398.83	1839398.832	0.00020041	1.278754
20	77787194.3	4930252629	18469747371	1831949.31	1831949.314	0.000200394	1.30103
21	74320876.4	5008039823	18391960177	1824817.05	1824817.047	0.000200379	1.322219
22	71159917.7	5082360700	18317639300	1817975.34	1817975.34	0.000200366	1.342423
23	68265288.9	5153520617	18246479383	1811400.75	1811400.754	0.000200353	1.361728
24	65604369.8	5221785906	18178214094	1805072.6	1805072.596	0.000200342	1.380211
25	63149682.3	5287390276	18112609724	1798972.51	1798972.506	0.000200331	1.39794
26	60877913.1	5350539958	18049460042	1793084.12	1793084.116	0.000200321	1.414973
27	58769151.4	5411417871	17988582129	1787392.77	1787392.775	0.000200312	1.431364
28	56806288.3	5470187023	17929812977	1781885.31	1781885.312	0.000200303	1.447158
29	54974539.3	5526993311	17873006689	1776549.85	1776549.846	0.000200295	1.462398
30	53261061.7	5581967850	17818032150	1771375.62	1771375.621	0.000200287	1.477121
31	51654644.9	5635228912	17764771088	1766352.87	1766352.869	0.00020028	1.491362
32	50145458.4	5686883557	17713116443	1761472.69	1761472.69	0.000200273	1.50515
33	48724845.3	5737029016	17662970984	1756726.95	1756726.954	0.000200267	1.518514
34	47385151.8	5785753861	17614246139	1752108.21	1752108.211	0.000200261	1.531479
35	46119585.2	5833139013	17566860987	1747609.62	1747609.624	0.000200255	1.544068
36	44922096.4	5879258598	17520741402	1743224.89	1743224.894	0.000200249	1.556303
37	43787280.1	5924180694	17475819306	1738948.21	1738948.212	0.000200244	1.568202

Table 6. Data Analysis for Fatigue Curve regarding Maximum Stress Verses Number of Cycle and Damage Factor Parameter Verses Number of Cycle

Assuming , Fatigue Damage Factor related to Surface Energy Microcracks i.e. A= 0.10 and Fatigue Kinematic Damage Factor (Crack Surface closed perfectly after unloading) i.e. β=0.15 at different Number of Cycle

N	$K	K	E	$stress	Maximum Stress	Resudial Strain	Log(N)
1	1282173402	0	23400000000	2080110.01	2080110.009	0.000205953	0
2	670041551	1282173402	22117826598	2073226.83	2073226.828	0.000203336	0.30103
3	458458692	1952214953	21447785047	2048829.13	2048829.131	0.000202387	0.47712125
4	350269622	2410673645	20989326355	2024987.1	2024987.103	0.000201886	0.60205999
5	284280732	2760943267	20639056733	2003480.66	2003480.658	0.000201572	0.69897
6	239711345	3045223999	20354776001	1984238.56	1984238.563	0.000201356	0.77815125
7	207530075	3284935345	20115064655	1966933	1966932.999	0.000201197	0.84509804
8	183169905	3492465419	19907534581	1951247.11	1951247.11	0.000201074	0.90308999
9	164069326	3675635324	19724364676	1936918.07	1936918.074	0.000200977	0.95424251
10	148678707	3839704650	19560295350	1923735.24	1923735.241	0.000200898	1
11	136004711	3988383357	19411616643	1911530.4	1911530.396	0.000200832	1.04139269
12	125380779	4124388068	19275611932	1900168.35	1900168.347	0.000200776	1.07918125
13	116342712	4249768847	19150231153	1889539.38	1889539.382	0.000200728	1.11394335
14	108557141	4366111560	19033888440	1879553.52	1879553.521	0.000200686	1.14612804
15	101778373	4474668701	18925331299	1870136.19	1870136.189	0.000200649	1.17609126
16	95821285.7	4576447074	18823552926	1861224.97	1861224.97	0.000200617	1.20411998
17	90543702.5	4672268359	18727731641	1852767.14	1852767.141	0.000200588	1.23044892
18	85834594.3	4762812062	18637187938	1844717.79	1844717.79	0.000200562	1.25527251
19	81605972.8	4848646656	18551353344	1837038.36	1837038.356	0.000200538	1.2787536
20	77787194.3	4930252629	18469747371	1829695.5	1829695.5	0.000200517	1.30103
21	74320876.4	5008039823	18391960177	1822660.2	1822660.202	0.000200497	1.32221929
22	71159917.7	5082360700	18317639300	1815907.05	1815907.052	0.000200479	1.34242268
23	68265288.9	5153520617	18246479383	1809413.68	1809413.677	0.000200463	1.36172784
24	65604369.8	5221785906	18178214094	1803160.27	1803160.274	0.000200447	1.38021124
25	63149682.3	5287390276	18112609724	1797129.24	1797129.236	0.000200433	1.39794001
26	60877913.1	5350539958	18049460042	1791304.83	1791304.833	0.00020042	1.41497335
27	58769151.4	5411417871	17988582129	1785672.96	1785672.959	0.000200408	1.43136376
28	56806288.3	5470187023	17929812977	1780220.91	1780220.914	0.000200396	1.44715803
29	54974539.3	5526993311	17873006689	1774937.22	1774937.222	0.000200385	1.462398
30	53261061.7	5581967850	17818032150	1769811.48	1769811.482	0.000200375	1.47712125
31	51654644.9	5635228912	17764771088	1764834.23	1764834.233	0.000200366	1.49136169
32	50145458.4	5686883557	17713116443	1759996.85	1759996.847	0.000200357	1.50514998
33	48724845.3	5737029016	17662970984	1755291.43	1755291.432	0.000200348	1.51851394
34	47385151.8	5785753861	17614246139	1750710.75	1750710.751	0.00020034	1.53147892
35	46119585.2	5833139013	17566860987	1746248.15	1746248.153	0.000200333	1.54406804
36	44922096.4	5879258598	17520741402	1741897.51	1741897.507	0.000200325	1.5563025
37	43787280.1	5924180694	17475819306	1737653.15	1737653.154	0.000200318	1.56820172
38	42710291.2	5967967974	17432032026	1733509.86	1733509.857	0.000200312	1.5797836
39	41686774.1	6010678266	17389321734	1729462.76	1729462.758	0.000200306	1.59106461
40	40712801.9	6052365040	17347634960	1725507.35	1725507.347	0.0002003	1.60205999

Table 7. Maximum Stress Observed by Peiyin Lu. Et al 2004 through Experiment

Smax	No. of Cycle	Factor	Max. Stress
0.85	1479000	2.2	158.4893192
0.84	1461600	2.3	199.5262315
0.75	1305000	4.1	12589.25412
0.74	1287600	4.4	25118.86432
0.69	1200600	5.05	112201.8454
0.685	1191900	5.2	158489.3192
0.68	1183200	5.75	562341.3252
0.65	1131000	6.2	1584893.192
0.63	1096200	6.3	1995262.315

Table 8. Damage predicted by Peiyin Lu. Et al 2004 through Experiment

Damage	Cyclic Ratio
0.16	0
0.17	0.02
0.172	0.04
0.18	0.05
0.19	0.08
0.195	0.1
0.2	0.12
0.21	0.145
0.22	0.198
0.23	0.24
0.24	0.255
0.25	0.28
0.26	0.32
0.28	0.37
0.3	0.395
0.31	0.43
0.36	0.52
0.4	0.585
0.48	0.68
0.55	0.72
0.7	0.88
0.88	1